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**CONTRACTS AND INSURANCE GROUP FORMATION BY  
MYOPIC PLAYERS**

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# Contracts and Insurance Group Formation by Myopic Players\*

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## Abstract

This paper employs a cooperative approach to insurance group formation problems. The insurance group formation is analyzed in terms of stability with respect to one-person deviations. Depending on the exact contractual setting, three stability concepts are proposed: individual, contractual and compensation stability. When we apply our general framework to the standard insurance setting of Rothschild and Stiglitz (1976), we find that, in each type of contractual setting, there are stable individually rational pooling outcomes while, on the contrary, individually rational separating outcomes are not stable.

**JEL Classification:** C71, D02

**Keywords:** Stability, Contracts, Group formation

## 1 Introduction

This paper views insurance group formation as a problem of cooperative nature. The value of an insurance group is generated by the ability of group members to smooth their consumption by pooling the risk of a loss. While insurance companies, organizations widely studied in the insurance market literature, have as an objective to maximize profits, the objective of an insurance group is to maximize the welfare of the group. In this setting the clients of an insurance group are also stakeholders of the company. As such insurance group clients divide the value of the group generated by their cooperation

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amongst themselves. The advantages of employing a cooperative approach to studying insurance organizations has been discussed by Boyd, Prescott, and Smith (1988).

The issue we investigate is the stability of endogenous insurance group formation under various contractual arrangements. We regard the insurance group<sup>1</sup> and an individual member as two sides in a contract. The arrangement gives particular rights, such as the right to end the contract and the right to be compensated to one or both of the parties. Depending on the allocation of rights, we distinguish between three types of contractual arrangements leading to the notions of *individual*, *contractual*, and *compensation* stability. Individual and contractual stability give the individual member the right to end the contract but do not allow for compensation rights. Contractual stability setting in addition gives a veto right to the group. To end the contract a member needs the agreement of the rest of the groups members. If they are worse-off after the change, they disagree. The third concept, compensation stability, gives both parties the right to end the contract and the right to be compensated in case they are worse-off after the change. Compensation stability has two complementary sides to it: pull-in and push-out stability. Pull-in stability reflects external stability by allowing an insurance group to attract new members. On the other hand, push-out stability reflects internal stability by allowing insurance groups to push out one of their members.

The focus on contractual arrangements is justified, on one hand, by the fact that contracts are common arrangements in insurance groups, but more importantly by the observation that agents, being part of large groups, are anonymous to each other. Thus, no group deviations are allowed in our setting. This makes the core an inadequate equilibrium concept for stability analysis. To the best of our knowledge all other works offering cooperative approach to insurance group formation, e.g. Demange and Guesnerie (2001)<sup>2</sup>, Kahn and Mookherjee (1995), Boyd, Prescott, and Smith (1988), and Boyd and Prescott (1986) allow for group deviations. These works focus on studying informational asymmetry and the properties of the core-stable insurance group structure under various channels of signalling. Instead we abstract from the asymmetric information problem to develop contract-based stability concepts.

With respect to the relation of our contractual stability concepts to the rest of the literature, individual and contractual stability are conceptually similar to individual and contractual stability introduced in the hedonic coalition formation setting (see Bogomolmaia and Jackson (2004)). In hedonic coalition formation, however, each player has exogenously determined preferences over membership in all coalitions while in our setting, a player's preferences are endogenous as they depend on her share from the group's value and her outside options.

To make this point more clear, we elaborate on the behavioral assumptions we make. We assume that each player makes a decision to join/leave a group based on her own perceived payoff without taking into account the effect of her actions on other players'

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<sup>1</sup>Here by group we mean the rest of the group members.

<sup>2</sup>Demange and Guesnerie (2001) explicitly model anonymous agents, however, this assumption only concerns the information channels rather than the group-wide deviation possibilities.

payoffs. A player's payoff from group membership is *endogenously* determined. It depends on her "power" to obtain a share of the group value. This power-based measure, though related, is not entirely determined by the player's marginal contribution<sup>3</sup> but also depends on her outside options, i.e. what other groups are being formed and how much she can possibly get as a share of their payoff, i.e. the *endogenous structure* of the insurance group formation. In this way a member of a group with high value might prefer to switch to a group with a smaller value because her payoff in the latter is higher than the share she gets from the former. By switching insurance groups, though, a player changes the outside options for the rest of the players, which she does not take into account at the time of the move. Accordingly, by switching insurance groups, the player also affects another player's relative power in bargaining for a share from the group value, not only in the two groups in which the membership has changed, but also in the rest of the insurance groups. Not taking into accounts the market-wide effects of her actions makes each player myopic.<sup>4</sup>

In this general setting, we find that there are insurance groups formation problems in which there is no individually stable outcomes. Moreover Pareto optimal outcomes may not be individually stable either. We use individual stability as a stepping stone for the construction of the other two stability concepts. Contractual stability and compensation stability have positive existence results. The contractual stability setting, however, does not allow for one-player value increasing deviations to take place, while compensation stability does.

As an application of our theoretical framework we consider the insurance market for damages as described in the seminal work by Rothschild and Stiglitz (1976). We refer to the setting as "*mutual insurance*" to emphasize the cooperative nature of the problem and distinguish it from the third-party market insurance setting studied by Rothschild and Stiglitz (1976). We focus on the stability of two types of outcomes widely studied in the literature: pooling and separating outcomes. We find that given the assumption of risk averse players, all pooling outcomes are contractually and compensation stable, while no separating outcomes are individually or compensation stable. The individually rational pooling outcomes are individually stable and, moreover, this type of outcomes exists in every mutual insurance formation problem. Finally, no individually rational separating outcome is contractually stable either. What drives these results is the possibility for side-payments within groups and, in the case of the compensation setting, between groups.

The paper is structured in the following way. In Section 2 we define the insurance formation problem and present the three stability concepts. We present existence results as well as a technical discussion on the relation between the concepts in Section 3. In Section 4 we apply the problem to the classical insurance market of Rothschild and Stiglitz (1976), and discuss the stable outcomes in terms of the risk composition of the

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<sup>3</sup>Insurance groups generate value by pooling the risk of their members. Thus, each member's contribution to the group value is completely determined by her ex-ante given risk type.

<sup>4</sup>The assumption of myopic players is in contrast to Kahn and Mookherjee (1995) who analyze players with a fair degree of "foresightedness".

groups. Section 5 concludes.

## 2 Insurance group formation and contractual settings

There is a finite set of players  $N = \{1, 2, \dots, n\}$ . Players in the set  $N$  form coalitions, to which we will refer as insurance groups. The collection of subsets of  $N$  is denoted by  $2^N$ . A player cannot be a member of more than one insurance group. The collection of all partitions of  $N$  is denoted as  $\mathcal{P}$ . Each insurance group generates value by the cooperation of its members. This value differs depending on the identity of the group members. For the purposes of the general analysis, it suffices to regard this relation as given by a value function  $v : 2^N \rightarrow \mathbb{R}$ . The pair  $(N, v)$  such that  $v(\emptyset) = 0$  defines an insurance group formation problem, to which we will refer as an insurance problem for brevity. Without loss of generality, we study zero-normalized problems, i.e.,  $v(\{i\}) = 0$  for all  $i \in N$ .

**Definition 2.1** An *outcome* of an insurance problem  $(N, v)$  is

- (i) a *partition*  $P \in \mathcal{P}$ ; and
- (ii) a *payoff vector*  $x \in \mathbb{R}^N$ , which is efficient, i.e.:

$$\sum_{i \in S} x_i = v(S), \text{ for all } S \in P.$$

We refer to outcomes  $(P, x)$  such that  $P$  has a maximum total partition value and  $x$  is efficient as *Pareto optimal* outcomes.

Given the insurance group value, a member's payoff depends on an endogenous allocation of the group value. A player  $i \in N$  prefers to be a member of an insurance group which yield a higher payoff to her. A fair player's payoff from an insurance group will also depend on the exact partition of the set of players, since the composition of the other groups in the partition defines the outside options. This point will become more clear with the discussion of the stability concepts below.

### 2.1 Individual stability

In the contractual arrangement of individual stability the right to end the contract is given only to the individual members and no compensatory obligations are imposed. Individual stability thus entails that a member cannot obtain a higher payoff by joining another insurance group in the partition or by forming a new single-member group.<sup>5</sup>

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<sup>5</sup>Implicit in the definition of individual stability is that a member can join an insurance group only if her membership is unanimously approved by the current group members. This is to say, a player can join an insurance group if the current members have at least as high payoff after she joins as they had before. This is why when a player decides on joining a coalition she bases her decision on her marginal contribution to the group value.

**Definition 2.2** An outcome  $(P, x)$  of an insurance problem  $(N, v)$  is **individually stable** if there are no  $i \in N$  and  $S \in P \cup \{\emptyset\}$  with  $i \notin S$  such that

$$x_i < v(S \cup \{i\}) - v(S).$$

The following example is used to illustrate the concept of individual stability.

**Example 2.3 [An individually stable outcome]** Let  $N = \{1, 2, 3\}$ . Consider the value function  $v$  as given below:

S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 2, 3\}$
$v(S)$	0	0	0	0	2	3	4	0

In the given insurance problem there is only one individually stable outcome  $(P, x)$ , namely  $P = \{\{1, 2, 3\}\}$  and  $x = 0$ . Clearly, this is an individually stable outcome: the players can deviate only by joining the empty set and obtain a payoff of zero.

No other outcome is individually stable because there is always at least one player who wants to deviate. As an example consider a Pareto optimal type of outcomes  $(P, x)$  given by  $P = \{\{1\}, \{2, 3\}\}$  and payoff vectors of the type  $x = (0, \alpha, 4 - \alpha)$  with  $\alpha \in [0, 4]$ . The best outside option for player 2 is to join 1 in insurance group  $\{1, 2\}$  where player 2's perceived payoff is at most two. Thus for player 2 not to have incentives to deviate it must be that  $x_2 = \alpha \geq 2$ . Similarly, for player 3 not to deviate, it must hold that  $x_3 = 4 - \alpha \geq 3$ . The two conditions cannot hold simultaneously in any efficient payoff vector and hence this type of outcomes cannot be individually stable. ♦

As seen above there are insurance problems in which no Pareto optimal outcome is individually stable. The next example shows that there are insurance problems in which there are no individually stable outcomes.

**Example 2.4 [No individually stable outcome]** Let  $N = \{1, 2, 3\}$ . Consider the value function  $v$  as given below:

S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 2, 3\}$
$v(S)$	0	0	0	0	2	3	4	-1

Note that the only difference with Example 2.3 is that here the grand coalition has a negative value. The outcome formed by the grand coalition and an efficient payoff vector cannot be individually stable: by the efficiency of the payoff vector follow that there is at least one player who has negative payoff, thus, such player will deviate by forming a new single-member group. ♦

## 2.2 Contractual stability

Contractual stability is based on individual incentives under the additional condition that a deviating member needs to acquire permission from the group in case she wants to

end the contract. The group grants the permission only if the rest of the group members are as well-off without that particular member as when she is part of the group. In a contractually stable outcome no player can obtain a higher payoff by joining another insurance group without making the members of her current insurance group worse-off.

**Definition 2.5** An outcome  $(P, x)$  of an insurance problem  $(N, v)$  is **contractually stable** if there are no  $i \in N$  and  $S, T \in P \cup \{\emptyset\}$  with  $i \in T$  and  $S \neq T$  such that

$$x_i < v(S \cup \{i\}) - v(S) \quad \text{and} \quad \sum_{j \in T \setminus \{i\}} x_j \leq v(T \setminus \{i\}).$$

The next example illustrates how contractual stability limits the deviating possibilities of a group member in contrast to individual stability.

**Example 2.6 [Contractually stable outcomes]** Let  $(N, v)$  be the insurance problem of Example 2.3.

There are multiple contractually stable outcomes in this insurance problem. Any partition besides the partition of singletons can be part of a contractually stable outcome. As an example, we study the following type of outcomes:  $P = \{\{1, 3\}, \{2\}\}$  and  $x = (\alpha, 0, 3 - \alpha)$  where  $\alpha \in (0, 3]$ . These are contractually stable outcomes: player 2's outside option is to join  $\{1, 3\}$  in the grand coalition but there her perceived payoff is negative. Neither player 1 nor 3 will give a permission to the other who has incentives to join player 2 since by being alone the player will have a strictly lower payoff.

An outcome that is not contractually stable is  $P = \{\{1, 3\}, \{2\}\}$  and  $x = (0, 0, 3)$ . Player 3's outside option to join player 2 in insurance group  $\{2, 3\}$  gives her a higher payoff than what she has, i.e.  $4 > 3 = x_3$ . Player 1 grants permission to player 3 to leave as she is indifferent between having a payoff of zero in the group  $\{1, 3\}$  or as a member of the singleton group  $\{1\}$ . ♦

The above example shows that in the contractual stability setting, there are situations in which a one-person deviation can lead to an increase in total partition value, yet, it is not performed because one of the parties of the contract is strictly worse-off. To overcome this restriction on profitable deviations, we allow for side payments between players after the contract between them has ended in the next contractual specification.

## 2.3 Compensation stability

First, the two complementary sides of compensation stability, pull-in stability and push-out stability, are introduced.

In the contractual setting of pull-in stability, the individual player is the only party who can end the contract. In case the remaining group members are worse-off, the new insurance group of the deviating member is obliged to compensate them for this loss. Thus, in a pull-in stable outcome there is no insurance group that by attracting a new member may increase its value enough to give higher payoffs to its members after possible compensation of the incoming player's previous insurance group.



**Definition 2.7** An outcome  $(P, x)$  of an insurance problem  $(N, v)$  is **pull-in stable** if there are no  $i \in N$  and  $S, T \in P$  with  $i \in T$  and  $S \neq T$  such that<sup>6</sup>

$$v(S) < v(S \cup \{i\}) - x_i - \max \left\{ 0, \sum_{j \in T \setminus \{i\}} x_j - v(T \setminus \{i\}) \right\}.$$

In push-out stability the right to end the contract is given only to the insurance group. An insurance group wants to end the contract of one of its members if by doing so, it can increase the payoffs of the remaining members. In the push-out setting a compensation is required in case the member whose contract has been ended has a lower best outside option than her current payoff as a group member.

**Definition 2.8** An outcome  $(P, x)$  of an insurance problem  $(N, v)$  is **push-out stable** if there are no  $i \in N$  and  $S, T \in P \cup \{\emptyset\}$  with  $i \in T$  and  $S \neq T$  such that

$$\sum_{j \in T \setminus \{i\}} x_j < v(T \setminus \{i\}) - \max \left\{ 0, x_i - (v(S \cup \{i\}) - v(S)) \right\}.$$

Combining pull-in and push-out stability, we have compensation stability.

**Definition 2.9** An outcome of an insurance problem is **compensation stable** if it is pull-in and push-out stable.

To illustrate that compensation setting may overcome the restriction on profitable deviations of the contractual stability setting, we consider the following example.

**Example 2.10 [Compensation stable outcomes]** Let  $(N, v)$  be as in Example 2.3. There is one type of compensation stable outcomes, namely the Pareto optimal type of outcomes  $(P, x)$  given by  $P = \{\{1\}, \{2, 3\}\}$  and  $x = (0, x_2, x_3)$  where  $x_2 + x_3 = 4$ . These outcomes are pull-in stable. Insurance group  $\{2, 3\}$  does not want to attract player 1 as a member since the grand coalition has lower value than their current value. Player 1 can increase the value of its insurance group by attracting either player 2 or 3. Yet, the increase is not enough to give her a higher payoff after she compensates the remaining member of group  $\{2, 3\}$  for the change.

These outcomes are also push-out stable. Neither player 2 nor 3 can increase her payoff by pushing the other player out to join player 1 in an insurance group and compensate her for the change, if needed. Player 1 cannot be pushed out of the singleton coalition either. ♦

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<sup>6</sup>Since the empty set is not regarded as an insurance group, it is not included in the possible set of coalitions that can pull a player in.

### 3 Existence and relations

The discussion of existence and relations between the stability contracts is focused on the notions of compensation and contractual stability. Example 2.4 shows that there are insurance problems with no individually stable outcomes.

From the definitions of the stability concepts the following results can be obtained in straightforward fashion.

**Proposition 3.1** *Let  $(N, v)$  be an insurance problem. Then the following results hold:*

- (i) *Any individually stable outcome is contractually stable;*
- (ii) *Any outcome  $(\{N\}, x)$  is pull-in stable;*
- (iii) *Any outcome  $(\{i\}_{i \in N}, 0)$  is push-out stable.*

We establish positive existence results with respect to compensation and contractual stability. In particular, all Pareto optimal outcomes are compensation and contractually stable.

**Theorem 3.2** *Any Pareto optimal outcome is both compensation and contractually stable.*

**Proof.** Let  $(N, v)$  be an insurance problem. Let  $(P^*, x)$  be a Pareto optimal outcome of this insurance problem.

*Compensation stability:* Suppose  $(P^*, x)$  is not a compensation stable outcome. Then either  $(P^*, x)$  is not pull-in stable or  $(P^*, x)$  is not push-out stable.

First, suppose  $(P^*, x)$  is not a pull-in stable outcome. Then there are  $i \in N$  and  $S, T \in P^*$  with  $i \in T$  and  $S \neq T$  such that

$$v(S) < v(S \cup \{i\}) - x_i - \max \left\{ 0, \sum_{j \in T \setminus \{i\}} x_j - v(T \setminus \{i\}) \right\}.$$

Using the efficiency of the payoff vector, the above inequality implies

$$v(S) + v(T) < v(S \cup \{i\}) + v(T \setminus \{i\}).$$

So the partition  $P = [P^* \setminus \{S, T\}] \cup \{S \cup \{i\}, T \setminus \{i\}\}$  has a higher total partition value contradicting the Pareto optimality of  $(P^*, x)$ .

Now suppose  $(P^*, x)$  is not a push-out stable outcome. Then there are  $i \in N$  and  $S, T \in P^* \cup \{\emptyset\}$  with  $i \in T$  and  $S \neq T$  such that

$$\sum_{j \in T \setminus \{i\}} x_j < v(T \setminus \{i\}) - \max \left\{ 0, x_i + (v(S \cup \{i\}) - v(S)) \right\}.$$

Using the efficiency of the payoff vector, the above inequality implies

$$v(T) + v(S) < v(T \setminus \{i\}) + v(S \cup \{i\}),$$

establishing a contradiction.

*Contractual stability:* Suppose  $(P^*, x)$  is not a contractually stable outcome. Then there are  $i \in N$  and  $S, T \in P^* \cup \{\emptyset\}$  with  $i \in T$  and  $S \neq T$  such that

$$\begin{aligned} x_i &< v(S \cup \{i\}) - v(S) \\ \sum_{j \in T \setminus \{i\}} x_j &\leq v(T \setminus \{i\}). \end{aligned}$$

Adding up the two inequalities and using the efficiency of the payoff vector, we find

$$v(T) + v(S) < v(T \setminus \{i\}) + v(S \cup \{i\}),$$

establishing a contradiction. ■

For establishing a relation between compensation stability and contractual stability we need to introduce one additional property.

**Definition 3.3** An outcome  $(P, x)$  of an insurance problem is individually rational if  $x_i \geq v(\{i\})$  for all  $i \in N$ .

**Proposition 3.4** Any compensation stable outcome which is individually rational is also contractually stable.

**Proof.** Let  $(N, v)$  be an insurance problem. Let  $(P, x)$  be a compensation stable outcome, which is individually rational. Then for all  $i \in N$  and  $S, T \in P$  with  $i \in T$  and  $S \neq T$ :

$$v(S) \geq v(S \cup \{i\}) - x_i - \max \left\{ 0, \sum_{j \in T \setminus \{i\}} x_j - v(T \setminus \{i\}) \right\}.$$

This implies that for all  $i \in N$  and  $S, T \in P$  with  $i \in T$  and  $S \neq T$

$$x_i \geq v(S \cup \{i\}) - v(S) \quad \text{or} \quad \sum_{j \in T \setminus \{i\}} x_j > v(T \setminus \{i\}).$$

Since  $(P, x)$  is individually rational  $x_i \geq 0 = v(\{i\}) - v(\{\emptyset\})$ . We conclude that for all  $i \in N$  and  $S, T \in P \cup \emptyset$  with  $i \in T$  and  $S \neq T$ , it holds that

$$x_i \geq v(S \cup \{i\}) - v(S) \quad \text{or} \quad \sum_{j \in T \setminus \{i\}} x_j > v(T \setminus \{i\}).$$

So,  $(P, x)$  is contractually stable. ■

Note that the proof of Proposition 3.4 in fact implies that any pull-in stable outcome which is individually rational is contractually stable.

## 4 Mutual insurance

We apply our stability concepts to the insurance problem studied by Rothschild and Stiglitz (1976). Their framework is adapted to the general setting presented in Section 2. We use the three types of stability concepts to analyze outcomes which differ in terms of the risk composition of the insurance group. In particular we discuss the pooling and separating outcomes which have received much attention in the literature. In Rothschild and Stiglitz (1976) framework, the separating type of outcomes is the only type that may be stable.

First, we describe the demand for insurance against financial loss. There is a finite set of players  $N$ . All players are expected utility maximizers each with the same increasing and strictly concave utility function  $u$  defined over monetary wealth  $y$ , i.e.  $u : \mathbb{R} \rightarrow \mathbb{R}$  with  $u' > 0$  and  $u'' < 0$ . Each player is endowed with an initial wealth  $w$ . With some probability  $\pi$  a player incurs a damage which has a monetary equivalent of  $D$  where  $D < w$ . There are two groups of players,  $L$  and  $H$ , forming a partition of  $N$ . Players in  $L$  have a low probability  $\pi_L$  of incurring a damage. Those in  $H$  have a high probability  $\pi_H$ . So  $\pi_L < \pi_H$ .

We assume that each insurance group offers the same amount of insurance to all members equalling the total damage. However, the group may require a different contribution fee per unit of insurance. The contribution fee  $q(S)$  of group  $S$  is determined by a zero-profit condition given by:

$$|S|q(S)D - \pi_L|S \cap L|D - \pi_H|S \cap H|D = 0.$$

Hence the contribution fee charged by a group  $S$  is

$$q(S) = \frac{|S \cap L|}{|S|}\pi_L + \frac{|S \cap H|}{|S|}\pi_H. \quad (1)$$

So, the contribution fee depends only on the relative size of the risk-pool of the insurance group. In particular,  $q(S) \in [\pi_L, \pi_H]$ . It is lowest when an insurance group consists of low-risk members only, and it is highest when it consists of high-risk members only.

The value of an insurance group  $S$  is defined to be the total utility of its members. Formally,

$$v(S) = |S| u(w - q(S)D) \quad \text{for all } S \in 2^N \setminus \{\emptyset\}. \quad (2)$$

*An insurance problem  $(N, v)$  derived from the above described tuple  $(L, H, u, w, D, \pi_L, \pi_H, q)$  with  $N = L \cup H$  and  $v$  defined by (2) is a mutual insurance group formation problem to which we refer as a mutual insurance problem for brevity. Note that the value function is not zero-normalized. However this affects neither the definitions of stability nor the results in Section 3.*

The next example is used to illustrate the mutual insurance setting.

**Example 4.1 [A mutual insurance problem]** Consider  $L = \{1, 2\}$  and  $H = \{3\}$ , so  $N = \{1, 2, 3\}$ . The probabilities of incurring a damage are given by  $\pi_L = 0.1$  and  $\pi_H = 0.7$ , respectively. Every player is endowed with the same amount of initial wealth  $w = 10$  while  $D = 9$ . Every player has preferences represented by the same increasing and strictly concave utility function  $u$  defined by  $u(y) = \sqrt{y}$  for  $y \geq 0$ .

The break-even contribution fee of each insurance group is calculated using Equation (1). The numbers are given below:

$$q(S) = \begin{cases} 0.1 & : S = \{1\}, \{2\}, \{1, 2\}; \\ 0.3 & : S = \{1, 2, 3\}; \\ 0.4 & : S = \{1, 3\}, \{1, 4\}; \\ 0.7 & : S = \{3\}. \end{cases}$$

Using the definition of group value given by Equation (2) and the break-even contribution fees, we derive the value function  $v$  as given below:

S	$\{\emptyset\}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 2, 3\}$
$v(S)$	0	3.0	3.0	1.9	6.0	5.0	5.0	8.1

In this mutual insurance problem all individually stable outcomes  $(P, x)$  are of the form  $P = \{\{1, 2, 3\}\}$ ,  $\sum_{i \in N} x_i = 8.1$  and  $x_1 \geq 3$ ,  $x_2 \geq 3$ ,  $x_3 \geq 1.9$ .

The contractual and compensation stable outcomes  $(P, x)$  coincide and are given by  $P = \{\{1, 2, 3\}\}$  and  $\sum_{i \in N} x_i = 8.1$ . ♦

In the rest of the section the analysis is focused on pooling<sup>7</sup> and separating types of outcomes. Below we give the formal definitions of these outcomes.

**Definition 4.2** An outcome  $(P, x)$  of a mutual insurance problem  $(N, v)$  is called pooling if  $P = \{N\}$ .

**Definition 4.3** An outcome  $(P, x)$  of a mutual insurance problem  $(N, v)$  with  $N = L \cup H$  is called separating if  $P = \{L, H\}$ .

We first show that the value function satisfies superadditivity.

**Lemma 4.4** Let  $(N, v)$  be a mutual insurance problem. Then for all  $S, T \in 2^N$  with  $S \cap T = \emptyset$

$$v(S) + v(T) \leq v(S \cup T).$$

**Proof.** Let  $(N, v)$  be a mutual insurance problem derived from  $(L, H, u, w, D, \pi_L, \pi_H, q)$  with  $N = L \cup H$  and  $v$  defined by Equation (2). Without loss of generality, consider

<sup>7</sup>Unlike Rothschild and Stiglitz (1976) we refer to pooling outcomes only as those outcomes which contain the grand coalition.

$S, T \in 2^N \setminus \{\emptyset\}$  with  $S \cap T = \emptyset$ . Then

$$\begin{aligned}
v(S) + v(T) &= |S|u\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H}{|S|}D\right) + |T|u\left(w - \frac{|T \cap L|\pi_L + |T \cap H|\pi_H}{|T|}D\right) \\
&= |S \cup T| \left\{ \frac{|S|}{|S \cup T|} u\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H}{|S|}D\right) + \frac{|T|}{|S \cup T|} u\left(w - \frac{|T \cap L|\pi_L + |T \cap H|\pi_H}{|T|}D\right) \right\} \\
&\leq |S \cup T| \left\{ u\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H + |T \cap L|\pi_L + |T \cap H|\pi_H}{|S \cup T|}D\right) \right\} \\
&= v(S \cup T).
\end{aligned}$$

The above inequality follows from the strict concavity of  $u$ . The inequality holds as equality in three cases:  $S, T \subset L$ ;  $S, T \subset H$ ; or  $|S| = |T|$  with  $q(S) = q(T)$ . ■

**Theorem 4.5** *In a mutual insurance problem we have*

- (i) *Any pooling outcome is compensation stable;*
- (ii) *Any pooling outcome is contractually stable;*
- (iii) *Any individually rational pooling outcome is individually stable;*
- (iv) *An individually rational pooling outcome exists.*

**Proof.** Lemma 4.4 implies that any pooling outcome is Pareto optimal. Therefore, (i) and (ii) follow from Theorem 3.2. (iii) is immediate from the definitions, while (iv) follows from the fact that  $v(N) \geq \sum_{i \in N} v(\{i\})$ , which is a consequence of Lemma 4.4. ■

Next, we discuss the stability of the separating outcomes.

**Theorem 4.6** *In a mutual insurance problem we have*

- (i) *No separating outcome is individually stable;*
- (ii) *No separating outcome is compensation stable;*
- (iii) *No individually rational separating outcome is contractually stable.*

**Proof.** Let  $(N, v)$  be a mutual insurance problem derived from  $(L, H, u, w, D, \pi_L, \pi_H, q)$  with  $N = L \cup H$  and  $v$  defined by Equation (2).

(i) Consider a separating outcome  $(\{L, H\}, x)$ . We will show that the value of the high-risk insurance group is insufficient to give each member at least her outside option

of joining the low-risk insurance group. For all  $i \in H$ , the outside option of joining the low-risk insurance group yields  $v(L \cup \{i\}) - v(L)$ . So for all  $i \in H$

$$\begin{aligned}
v(H) - |H|(v(L \cup \{i\}) - v(L)) &= \\
&= |H|u(w - \pi_H D) - |H|\left((|L| + 1)u\left(w - \frac{|L|\pi_L + \pi_H}{|L| + 1}D\right) - |L|u(w - \pi_L D)\right) \\
&= |H|(|L| + 1)\left\{\frac{1}{|L| + 1}u(w - \pi_H D) + \frac{|L|}{|L| + 1}u(w - \pi_L D) - u\left(w - \frac{|L|\pi_L + \pi_H}{|L| + 1}D\right)\right\} \\
&< 0.
\end{aligned}$$

Here, the inequality follows from the strict concavity of  $u$ .

(ii) Consider a separating outcome  $(\{L, H\}, x)$ . We will first show that total partition value increases when a high-risk player joins the low-risk group.

For any  $i \in H$

$$\begin{aligned}
v(L \cup \{i\}) + v(H \setminus \{i\}) - (v(L) + v(H)) &= \\
&= (|L| + 1)u\left(w - \frac{|L|\pi_L + \pi_H}{|L| + 1}D\right) - u(w - \pi_H D) - |L|u(w - \pi_L D) \\
&= (|L| + 1)\left\{u\left(w - \frac{|L|\pi_L + \pi_H}{|L| + 1}D\right) - \frac{1}{|L| + 1}u(w - \pi_H D) - \frac{|L|}{|L| + 1}u(w - \pi_L D)\right\} \\
&> 0.
\end{aligned}$$

The above inequality follows from the strict concavity of  $u$ .

Using the efficiency of the payoffs we have

$$v(L \cup \{i\}) + v(H \setminus \{i\}) - (v(L) + x_i + \sum_{j \in H \setminus \{i\}} x_j) > 0.$$

The above inequality implies that at least one of the inequalities below holds for any  $i \in H$

$$(v(L \cup \{i\}) - v(L) - x_i) > 0 \quad \text{or} \quad v(H \setminus \{i\}) - \sum_{j \in H \setminus \{i\}} x_j > 0.$$

So for any  $i \in H$ ,

$$\begin{aligned}
v(L) &< v(L \cup \{i\}) - x_i - \max\left\{0, \sum_{j \in H \setminus \{i\}} x_j - v(H \setminus \{i\})\right\} \\
\text{or} \\
\sum_{j \in H \setminus \{i\}} x_j &< v(H \setminus \{i\}) - \max\left\{0, x_i - (v(L \cup \{i\}) - v(L))\right\}.
\end{aligned}$$

Therefore, at least one of the pull-in and push-out conditions is violated.

(iii) Consider an individually rational separating outcome  $(\{L, H\}, x)$ . By individual rationality, for all  $i \in H$

$$x_i \geq v(\{i\}) = u(w - \pi_H D).$$

By the efficiency of the payoff vector, for all  $i \in H$

$$\sum_{i \in H} x_i = v(H) = |H|u(w - \pi_H D).$$

Combining both conditions, we obtain that  $x_i = u(w - \pi_H D)$  for all  $i \in H$ .

Hence, if any player wants to leave the high risk-group, the rest of the members will grant permission: for all  $i \in H$

$$\sum_{j \in H \setminus \{i\}} x_j = |H - 1|u(w - \pi_H D) = v(H \setminus \{i\}).$$

To show that this outcome is not contractually stable, we need to show that the value of the high-risk group is insufficient to give all of its members their best outside option, i.e. what they can get by joining the low-risk insurance group. This is already shown in the proof of (i) above. ■

## 5 Conclusion

We studied insurance group formation problems with anonymous membership which allows for disregarding group deviations. To analyze the stability of group formation in such settings we employ a contract-based approach. We defined three contractual arrangements which led to the notions of individual, contractual, and compensation stability. We have seen that individually stable outcomes may not always exist. However, if they do, they are necessarily individually rational. Contractual and compensation stable outcomes, can be found in any insurance group formation problem. However, not all of these outcomes may be individually rational.

We applied this general framework to a Rothschild and Stiglitz (1976) type of model. Given the possibilities of side payments we find that any pooling outcome is Pareto optimal. An individually rational pooling outcome is individually, contractually and compensation stable. An individually rational separating outcome, however, is not stable in any contractual setting.

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